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Significantly super-Chandrasekhar limiting mass white dwarfs as progenitors for peculiar over-luminous type Ia supernovae

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Since 2012, we have initiated developing systematically the simplistic to rigorous models to prove that highly super-Chandrasekhar, as well as highly sub-Chandrasekhar, limiting mass white dwarfs are possible to exist. We show that the mass of highly magnetized or modified Einstein's gravity induced white dwarfs could be significantly super-Chandrasekhar and such white dwarfs could altogether have a different mass-limit. On the other hand, type Ia supernovae (SNeIa), a key to unravel the evolutionary history of the universe, are believed to be triggered in white dwarfs having mass close to the Chandrasekhar-limit. However, observations of several peculiar, over- and under-luminous SNeIa argue for exploding masses widely different from this limit. We argue that explosions of super-Chandrasekhar limiting mass white dwarfs result in over-luminous SNeIa. We arrive at this revelation, first by considering simplistic, spherical, Newtonian white dwarfs with constant magnetic fields. Then we relax the Newtonian assumption and consider the varying fields, however obtain similar results. Finally, we consider a full scale general relativistic magnetohydrodynamic description of white dwarfs allowing their self-consistent departure from a sphere to ellipsoid. Subsequently, we also explore the effects of modified Einstein's gravity. Our finding questions the uniqueness of the Chandrasekhar-limit. It further argues for a possible second standard candle, which has many far reaching implications.

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1 Introduction

Since last 2 – 3 years, we have been exploring physics behind the origin of peculiar, over-luminous type Ia supernovae (SNeIa) by invoking super-Chandrasekhar white dwarfs as their progenitor. Since our proposal, idea of super-Chandrasekhar white dwarfs has come into lime-light — so many follow-up papers have appeared subsequently. It was indeed argued earlier, based on observation, that such peculiar, over-luminous SNeIa would exhibit super-Chandrasekhar progenitors [1].

The peculiarity of over-luminous SNeIa not only lies with their power, but also with the shape of their lightcurves. All SNeIa generally follow a luminosity-stretch relation — Philip’s relation [2] — larger the peak luminosity, slower is the declining rate of the power and vice versa. Such a relation does not hold in peculiar SNeIa — the larger peak luminosity corresponds to the faster declining rate of power. Interestingly, the kinetic energy of these SNIa ejecta is observed to be very low, hence higher power in the lightcurve could only be explained by invoking a larger progenitor mass.

Any new idea, when proposed, generally is tested with a simplistic model first. Once, the results based on a simplistic model show promise to explain observations and/or experiments, then more realistic self-consistent models, introducing more sophisticated physics, are introduced in order to fine-tune the original model. Without being an exception, we have also followed the same tactics to develop our super-Chandrasekhar white dwarf model.

We have, so far, approached towards this mission through the following steps. First, we have considered most simplistic, spherically symmetric, highly magnetized white dwarfs in the Newtonian framework, assuming the magnetic field to be constant or almost constant throughout (or modeling, as if, the inner region of white dwarfs) [3]. However, it has been speculated in that work itself that the self-consistent consideration of deformation of white dwarfs would reveal a similar super-Chandrasekhar mass at lower fields. In the same model framework, we have also shown that magnetized white dwarfs altogether have a new mass-limit, 80% larger than the Chandrasekhar-limit [4], in the same spirit as the Chandrasekhar-limit was obtained [5]. Afterwards, we have removed both the assumptions: the Newtonian description and spherical symmetry (e.g. [6, 7]). Note that magnetized white dwarfs could be significantly smaller in size compared to their conventional counter-parts [3, 4] and, hence, general relativistic effects therein may not be negligible. Thus, based on a full scale general relativistic magnetohydrodynamic (GRMHD) description [8], we have explored more self-consistent white dwarfs which are ellipsoid and have revealed similar stable masses as obtained in the simpler framework [7].

All the above explorations are, however, based on the power of magnetic fields. We have also explored the effects of a possible modification to Einstein’s gravity. Note that apart from over-luminous ones, some of SNeIa are under-luminous as well. By invoking the modified Einstein’s gravity (in the first instance, simplistic Starobinsky

gravity), we have shown that, depending on their density, white dwarfs could have significantly super- as well as sub-Chandrasekhar limiting masses. This apparently unifies two apparently disjoint classes of SNeIa and argues that Einstein's gravity theory may not be the ultimate theory even in the stellar physics.

In the next sections, we describe all the explorations one by one to firmly establish our theory. Our attempt will be to demonstrate significantly super- and sub-Chandrasekhar limiting masses, rather than their explosions to give rise to the peculiar, respectively, over- and under-luminous SNeIa. We will assume that such white dwarfs, on approaching their respective super- and sub-Chandrasekhar limiting masses, will reveal over- and under-luminous SNeIa respectively, whose proof will be deferred for a future work.

2 Model I: Spherical white dwarfs with constant magnetic fields in the Newtonian framework

In the presence of a strong uniform magnetic field, the energy states of a free electron are quantized into Landau orbitals. Theoretically, the Landau quantization effects start affecting electrons at a field $B_c = 4.414 \times 10^{13} \text{G}$ and above, although practically it requires at least another order of magnitude higher field to affect them.

The Fermi energy level (E_F) of a Landau quantized electron is given by

$$E_F^2 = p_F(\nu)^2 c^2 + m_e^2 c^4 (1 + 2\nu B_D), \quad (1)$$

when $p_F(\nu)$ is the Fermi momentum of ν th Landau level ($\nu = 0, 1, 2, \dots$), m_e the mass of electrons, c the speed of light, B_D the magnetic field in units of B_c . The Fermi energy of electrons in units of $m_e c^2$ for a given ν is given by

$$\epsilon_F^2 = x_F(\nu)^2 + (1 + 2\nu B_D), \quad (2)$$

where $x_F(\nu)$ is the Fermi momentum in units $m_e c$. As $x_F(\nu)^2 \geq 0$, the maximum number of occupied Landau levels

$$\nu_m = \left(\frac{\epsilon_{Fmax}^2 - 1}{2B_D} \right)_{\text{nearest lowest integer}}. \quad (3)$$

For an one Landau level system, when only ground Landau level ($\nu = 0$) is occupied, $\nu_m = 1$. Similarly, for a two level system, when ground ($\nu = 0$) and first ($\nu = 1$) levels are occupied, $\nu_m = 2$, and so on.

We write [9] the electron number density

$$n_e = \frac{2B_D}{(2\pi)^2 \lambda_e^3} \sum_{\nu=0}^{\nu_m} g_\nu x_F(\nu), \quad (4)$$

where the Compton wavelength of the electron $\lambda_e = \hbar/m_e c$ and g_ν is the degeneracy that arises due to the Landau level splitting, such that, $g_\nu = 1$ for $\nu = 0$ and $g_\nu = 2$ for $\nu \geq 1$, the matter density

$$\rho = \mu_e m_H n_e, \quad (5)$$

where μ_e is the mean molecular weight per electron and m_H the mass of hydrogen atom, the electron degeneracy pressure

$$P = \frac{2B_D}{(2\pi)^2 \lambda_e^3} m_e c^2 \sum_{\nu=0}^{\nu_m} g_\nu (1 + 2\nu B_D) \eta \left(\frac{x_F(\nu)}{(1 + 2\nu B_D)^{1/2}} \right), \quad (6)$$

where

$$\eta(y) = \frac{1}{2} y \sqrt{1 + y^2} - \frac{1}{2} \ln(y + \sqrt{1 + y^2}). \quad (7)$$

In the limit $E_F \gg m_e c^2$, which corresponds to a very high density (as well as a high magnetic field and, hence, $\nu_m = 1$), combining equations (4), (5) and (6) we obtain

$$P = \frac{m_e c^2}{2Q\mu_e m_H} \rho^2 = K_m \rho^\Gamma, \quad (8)$$

which corresponds to the polytropic EoS with $\Gamma = 2$.

The underlying magnetized, spherical white dwarf obeys the magnetostatic equilibrium condition

$$\frac{1}{\rho + \rho_B} \frac{d}{dr} \left(P + \frac{B^2}{8\pi} \right) = F_g + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi(\rho + \rho_B)} \Big|_r, \quad (9)$$

when r is the radial distance from the center of white dwarf, \vec{B} the magnetic field in G, $B^2 = \vec{B} \cdot \vec{B}$, F_g the radial component of gravitational force and ρ_B the magnetic density. This equation is supplemented by the equation for the estimate of mass (M) within any r , given by

$$\frac{dM}{dr} = 4\pi r^2 (\rho + \rho_B). \quad (10)$$

The magnetic field in the white dwarf is assumed to be very slowly varying such that the combined effect of magnetic pressure gradient and tension is cancelled out by the effect of gravity due to the magnetic density (as justified previously [11, 12]). Moreover, at a very large density (in the limiting case considered here in the spirit of the Chandrasekhar-limit), the star becomes so compact as if the magnetic field remains (almost) constant throughout. Hence, taking above facts into consideration (which effectively converts the magnetostatic balance condition to hydrostatic condition) and combining equations (9) and (10), we obtain

$$\frac{1}{r^2} \frac{d}{dr} \left(\frac{r^2}{\rho} \frac{dP}{dr} \right) = -4\pi G \rho, \quad (11)$$

where G is the Newton's gravitation constant. Let us define

$$\rho = \rho_c \theta^n, \quad r = a\xi, \quad (12)$$

where ρ_c is the central density of the white dwarf, θ a dimensionless variable, $n = 1/(\Gamma - 1)$, ξ another dimensionless variable and constant a carries the dimension of length defined as

$$a = \left[\frac{(n+1)K_m \rho_c^{\frac{1-n}{n}}}{4\pi G} \right]^{1/2}. \quad (13)$$

Thus using equations (8), (12) and (13), equation (11) reduces to

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n, \quad (14)$$

which is the famous Lane-Emden equation. Equation (14) can be solved for a given n , along with the boundary conditions

$$\theta(\xi = 0) = 1, \quad \left(\frac{d\theta}{d\xi} \right)_{\xi=0} = 0. \quad (15)$$

Note that the surface of white dwarf corresponds to $\xi = \xi_1$ when $\theta = 0$, such that its radius

$$R = a\xi_1. \quad (16)$$

Also by combining equations (10), (12) and (14), we obtain the mass of the white dwarf

$$M = 4\pi a^3 \rho_c \int_0^{\xi_1} \xi^2 \theta^n d\xi. \quad (17)$$

Now, the scalings of mass and radius of the white dwarf with ρ_c are easily obtained as

$$M \propto K_m^{3/2} \rho_c^{(3-n)/2n}, \quad R \propto K_m^{1/2} \rho_c^{(1-n)/2n}. \quad (18)$$

Clearly $n = 3$ ($\Gamma = 4/3$) corresponds to M independent of ρ_c , provided K_m is independent of ρ_c , and hence limiting mass.

However, in the extreme condition of magnetized white dwarfs, from equations (3) and (8), we obtain

$$K_m = K \rho_c^{-2/3}, \quad (19)$$

where K is a constant and hence from equation (18)

$$M \propto \rho_c^{3(1-n)/2n}, \quad R \propto \rho_c^{(3-5n)/6n}, \quad (20)$$

revealing M independent of ρ_c for $n = 1$, when the radius becomes independent of the mass in the mass-radius relation [10, 4]. Now combining equations (13), (17) with $n = 1$, we obtain the value of limiting mass

$$M_l = \frac{5.564}{\mu_e^2} \left(\frac{c\hbar}{Gm_H^{4/3}} \right)^{3/2}, \quad (21)$$

when the limiting radius $R_l \rightarrow 0$. For $\mu_e = 2$, $M_l = 2.58M_\odot$, where M_\odot is solar mass. Importantly, for finite but high density and magnetic field, e.g. $\rho_c = 2 \times 10^{10} \text{ gm/cc}$ and $B = 8.8 \times 10^{15} \text{ G}$ when $E_{Fmax} = 20m_e c^2$, $M = 2.44M_\odot$ and R is about 650km. Note that above ρ_c and B are below their respective upper limits set by the instabilities of pycnonuclear fusion, inverse- β decay and general relativistic effects [12], however still produce a significantly super-Chandrasekhar smaller white dwarf with a mass very close to the limiting value.

3 Model II: Spherical white dwarfs with varying magnetic fields in the general relativistic framework

It is important to note that a large number of white dwarfs have been discovered by the Sloan Digital Sky Survey (SDSS), having high surface fields $10^5 - 10^9 \text{ G}$ [13, 14]. It is likely that the observed surface field is several orders of magnitude smaller than the central field. Thus, it is important to perform investigations, in the presence of varying fields, by accounting dP_B/dr and ρ_B , in addition to dP/dr . Keeping this in mind, we model the variation of the magnitude of magnetic field as a function of ρ [15] given by

$$B \left(\frac{\rho}{\rho_0} \right) = B_s + B_0 \left[1 - \exp \left(-\eta \left(\frac{\rho}{\rho_0} \right)^\gamma \right) \right], \quad (22)$$

where ρ_0 , for the present purpose, is chosen to be one-tenth of ρ_c of the corresponding white dwarf, B_s the surface magnetic field, B_0 a parameter having dimension of B , η and γ are dimensionless parameters which determine how exactly the field magnitude decays from the center to the surface. We consider the cases with $10^9 \text{ G} \leq B_s \leq 10^{12} \text{ G}$. However, for the central magnetic field $B_{\text{cent}} \geq 10^{14} \text{ G}$, the result is independent of the above considered B_s or less.

In the general relativistic framework, the spherically symmetric white dwarfs are described by the (magnetized) Tolman-Oppenheimer-Volkoff (TOV) equations [6, 16], given by

$$\frac{dM(r)}{dr} = 4\pi r^2 (\rho(r) + \rho_B), \quad (23)$$

$$\frac{d\rho(r)}{dr} = -\frac{G\left(\rho(r) + \rho_B + \frac{(P(r)+P_B)}{c^2}\right)\left(M(r) + \frac{4\pi r^3(P(r)+P_B)}{c^2}\right)}{r^2\left(1 - \frac{2GM(r)}{c^2 r}\right)\left(\frac{dP(r)}{d\rho} + \frac{dP_B}{d\rho}\right)}, \quad (24)$$

assuming the averaged effects of magnetic field to the pressure and hence the star is isotropic such that the magnetic pressure $P_B = B^2/24\pi$. Validity of such a consideration has been already justified earlier [12, 17, 18, 19].

In the presence of a strong magnetic field, the total pressure of the system in the direction perpendicular to the field is $P_\perp = P + B^2/8\pi$ (neglecting magnetization, which is much smaller compared to $B^2/8\pi$ for the field of present interest), while that in the direction parallel to the field is given by $P_\parallel = P - B^2/8\pi$. Hence, we propose two plausible constraints on the magnetic field profile in order to avoid any instability due to negative parallel pressure, such that: (i) the average parallel pressure, given by $P - P_B$, should remain positive throughout the white dwarf, (ii) P_\parallel should remain positive throughout the white dwarf (see [6] for details). Various results obtained by considering different magnetic field profiles, the number of occupied Landau levels at the center ν_{mc} , and E_{Fmax} , are summarized in Figure 1. We restrict E_{Fmax} to $50m_e c^2$, in order to avoid possible neutronization of the matter.

Figure 1(d) depicts the most important results, i.e. the mass-radius relations. Note that the radii of the white dwarfs having $E_{Fmax} = 50m_e c^2$ are more than a factor of two smaller than those with $E_{Fmax} = 20m_e c^2$ for roughly the same range of M . This is expected because a higher E_{Fmax} implies a higher ρ_c and hence more compact objects. For example, for the cases pertaining to the constraint (i), $M_{\max} = 3.33M_\odot$ and $R = 1605\text{km}$ for $E_{Fmax} = 20m_e c^2$ ($\rho_c = 1.55 \times 10^{10}\text{gm/cc}$), while $M_{\max} = 3.28M_\odot$ and $R = 670\text{km}$ for $E_{Fmax} = 50m_e c^2$ ($\rho_c = 2.42 \times 10^{11}\text{gm/cc}$). Similarly, for the cases pertaining to the constraint (ii), $M_{\max} = 2.1M_\odot$ and $R = 1237\text{km}$ for $E_{Fmax} = 20m_e c^2$, while $M_{\max} = 2.06M_\odot$ and $R = 512\text{km}$ for $E_{Fmax} = 50m_e c^2$.

4 Model III: Most self-consistent spheroidal white dwarfs with varying magnetic fields in the general relativistic framework

Now we explore, using the (modified) *XNS* code [8, 20], GRMHD analyses of magnetized, rotating white dwarfs and confirm the validity of all the preceding results. Although, originally the *XNS* code was developed to investigate deformed neutron stars, recently it was modified [7, 21] in order to obtain equilibrium configurations of deformed, magnetized, rotating white dwarfs. We self-consistently find that for a range $10^{10} \leq \rho_c \leq 10^{11}\text{gm/cm}^3$, the maximum magnetic field strength inside the white dwarf ranges as $10^{13} \leq B_{\max} \leq 10^{15}\text{G}$. Consequently, $\nu_m \geq 20$ for this range of ρ_c and B_{\max} , which does not significantly modify Γ (see, e.g., [3]). We consider

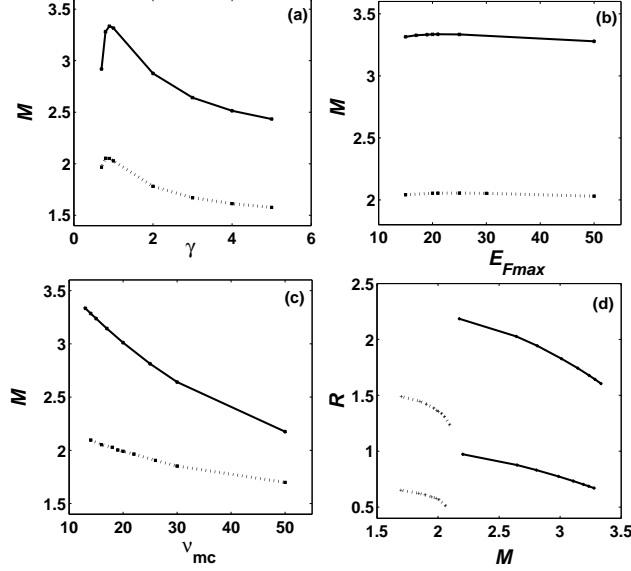


Figure 1: Super-Chandrasekhar white dwarfs with varying magnetic fields — the solid and dotted lines represent the cases corresponding to constraints (i) and (ii) respectively (see text). (a) M as a function of γ for $E_{Fmax} = 20m_e c^2$, $B_{cent} = 6.77 \times 10^{14} \text{G}$, maximum η , upto which constraints (i) and (ii) satisfy, for respective γ s. (b) M as a function of E_{Fmax} for $B_{cent} = 6.77 \times 10^{14} \text{G}$, $\gamma = 0.9$ for solid line and $B_{cent} = 5.18 \times 10^{14} \text{G}$, $\gamma = 0.8$ for dotted line, for the respective best η s. (c) M as a function of ν_{mc} for $E_{Fmax} = 20m_e c^2$, $\gamma = 0.9$ for solid line and $\gamma = 0.8$ for dotted line, for the respective best η s. (d) The topmost solid and dotted lines represent the M - R relations corresponding to (c), while the solid and dotted lines at the bottom represent the M - R relations corresponding to (c) but with $E_{Fmax} = 50m_e c^2$. M , R and E_{Fmax} are in units of M_\odot , 1000km and $m_e c^2$ respectively.

for all the computations $\rho_c = 1.9902 \times 10^{10} \text{gm/cc}$, which is high enough to ensure almost complete relativistic electron degeneracy, so that we may use the polytropic EoS with $n = 3$ (i.e. $\Gamma = 4/3$) consistently throughout the star. This density is still lower than the limit at which gravitational instabilities in general relativity set in, which is about $3 \times 10^{10} \text{gm/cc}$.

We refer the readers to the previous works [8, 7, 20, 21] for a complete description of GRMHD formulation, which includes the equations characterizing the geometry of the magnetic field and the underlying current distribution, as well as the numerical technique employed by the *XNS* code to solve them. As mentioned earlier, the presence of a strong magnetic field in a compact object generates an anisotropy in the magnetic pressure which in turn causes the star to be deformed, which we consider here self-consistently. Of course, the degree of anisotropy depends on the strength and geometry of the magnetic field. We construct axisymmetric white dwarfs in spherical

$B_{\max}(10^{14}\text{G})$	$M(M_{\odot})$	$r_e(\text{km})$	$\Omega_{eq}(\text{sec}^{-1})$	KE/GE	ME/GE	r_p/r_e
0	1.769	1410	2.990	0.126	0	0.613
2.299	1.959	1676	2.180	0.132	0.046	0.603
2.996	2.318	2171	1.339	0.136	0.108	0.583
3.584	3.159	3322	0.593	0.132	0.203	0.584

Table 1: Differentially rotating configurations with purely toroidal magnetic field, with changing B_{\max} and $\Omega_c = 30.42\text{sec}^{-1}$ fixed. KE/GE and ME/GE are the ratios of kinetic and gravitational energies and magnetic to gravitational energies respectively, r_p is the polar radius.

polar coordinates (r, θ, ϕ) , self-consistently accounting for the deviation from spherical symmetry. Stationary configurations can have many rotation laws consistent with the magnetostatic balance condition.

Figure 2 illustrates a typical set of GRMHD solutions for magnetized, differentially rotating white dwarfs. With the increase of toroidally dominated magnetic field, upto the maximum value considered $B_{\max} = 3.584 \times 10^{14}\text{G}$ (do not confuse with B_{\max} in a given white dwarf defined in Model II), the mass increases up to $3.159M_{\odot}$ — by nearly 78% from the mass in the non-magnetised case. The equatorial radius r_e for B_{\max} becomes about 3322km, more than double that of the non-magnetized configuration. The surface angular velocity Ω_{eq} reduces from 2.99 to 0.593sec^{-1} . This could be understood as the increase of r_e increases the centrifugal force at the equator for a given Ω_{eq} , which enforces decreased Ω_{eq} in order to obtain equilibrium solutions. See Table 1 for details. We furthermore note the polar concavities developing primarily due to the differential rotation, are accentuated by the toroidal field. Note that chosen toroidally dominated fields here have already been shown to be stable and realistic based on the prescription for twisted-torus geometries [22]. However, the poloidally dominated magnetized stable white dwarfs have also been shown to be possible with much smaller radii [21], $< 1000\text{km}$.

5 Model IV: Spherical non-rotating, non-magnetized white dwarfs in the framework of modified Einstein's gravity

Let us consider the 4-dimensional action as

$$S = \int \left[\frac{1}{16\pi} f(R) + \mathcal{L}_M \right] \sqrt{-g} d^4x, \quad (25)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, \mathcal{L}_M the Lagrangian density of the matter field, R the scalar curvature defined as $R = g^{\mu\nu} R_{\mu\nu}$, when $R_{\mu\nu}$ is the Ricci

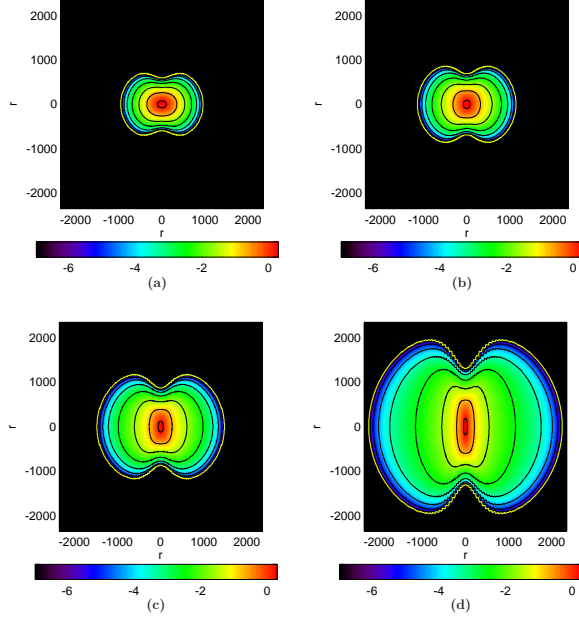


Figure 2: Sequence of differentially rotating white dwarfs with a purely toroidal magnetic field, with changing B_{\max} and fixed $\Omega_c = 30.42\text{sec}^{-1}$. The panels are contour plots of $\log(\frac{\rho}{\rho_0})$, with $\rho_0 = 10^{10}\text{gm/cc}$, corresponding to the $B_{\max}/10^{14}\text{G}$ values (a) 0, (b) 2.299, (c) 2.996, (d) 3.584. The corresponding physical quantities are listed in Table 1.

tensor and f an arbitrary function of R (in Einstein's gravity $f(R) = R$). For the present purpose, we choose the Starobinsky model [23] defined as $f(R) = R + \alpha R^2$, when α is a constant such that $\alpha R \ll 1$, revealing only the first order correction and $g_{\mu\nu} = g_{\mu\nu}^0 + \alpha g_{\mu\nu}^1$, when $g_{\mu\nu}^0$ is the metric tensor in Einstein's gravity. However, similar effects could also be obtained in other modified gravity theories, e.g. Born-Infeld gravity (e.g. [24]). Now, on extremizing the above action, one obtains the modified field equation as

$$G_{\mu\nu} + \alpha \left[2RG_{\mu\nu} + \frac{1}{2}R^2 g_{\mu\nu} - 2(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\mu \nabla^\mu) R \right] = 8\pi T_{\mu\nu}, \quad (26)$$

where $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}R/2$, is Einstein's field tensor, $T_{\mu\nu}$ the energy-momentum tensor of the matter field and ∇_μ the covariant derivative.

Furthermore, we consider the hydrostatic equilibrium condition: $g_{\nu r} \nabla_\mu T^{\mu\nu} = 0$, with zero velocity. Hence, we obtain the *modified* TOV equations as the differential equations for mass $M_\alpha(r)$, pressure $P_\alpha(r)$ (or density $\rho_\alpha(r)$) and gravitational potential $\phi_\alpha(r)$, of spherically symmetric white dwarfs (see [25] for details).

The boundary conditions for the solutions of modified TOV equations are $M_\alpha(0) = 0$ and $\rho_\alpha(0) = \rho_c$. The chosen EoS is same as that chosen for nonmagnetized white dwarfs [5], described in Model I, given by $P_0 = K\rho_0^{1+(1/n)}$, for extremely low and high densities, where P and ρ of Model I are replaced by P_0 and ρ_0 respectively in the spirit of linear perturbation.

Figure 3 shows that with the increase of α , the region of overlap of curves with the curve of Einstein's gravity ($\alpha = 0$) recedes to a lower ρ_c . Modified Einstein's gravity effects become important and visible at $\rho_c \geq 10^8$, 4×10^7 and $2 \times 10^6 \text{ gm/cc}$, for $\alpha = 2 \times 10^{13}$, 8×10^{13} and 10^{15} cm^2 respectively. For a given α , with the increase of ρ_c , M_α first increases, reaches a maximum and then decreases, like the $\alpha = 0$ case. With the increase of α , maximum mass M_{max} decreases and for $\alpha = 10^{15} \text{ cm}^2$ it is highly sub-Chandrasekhar ($0.81M_\odot$). This feature of the modified gravity effect in white dwarfs was completely overlooked earlier. In fact, M_{max} for all the chosen $\alpha > 0$ is sub-Chandrasekhar, ranging $1.31 - 0.81M_\odot$. This is a remarkable finding since it establishes that even if ρ_c s for these sub-Chandrasekhar white dwarfs are lower than the conventional value at which SNeIa are usually triggered, an attempt to increase the mass beyond M_{max} with increasing ρ_c will lead to a gravitational instability. This presumably will be followed by a runaway thermonuclear reaction, provided the core temperature increases sufficiently due to collapse. Occurrence of such thermonuclear runaway reactions, triggered at densities as low as 10^6 gm/cc , has already been demonstrated [26]. Thus, once M_{max} is approached, a SNIa is expected to trigger just like in the $\alpha = 0$ case, explaining the under-luminous SNeIa [27, 28], like SN 1991bg mentioned above.

For $\alpha < 0$ cases, Fig. 3 shows that for $\rho_c > 10^8 \text{ gm/cc}$, the $M_\alpha - \rho_c$ curves deviate from the general relativity and M_{max} for all the three cases corresponds to $\rho_c = 10^{11} \text{ gm/cc}$, which is an upper-limit chosen to avoid possible neutronization. Interestingly, all values of M_{max} are highly super-Chandrasekhar, ranging $1.8 - 2.7M_\odot$. Thus, while the general relativity effect is very small, modified Einstein's gravity effect could lead to $\sim 100\%$ increase in the limiting mass of white dwarfs, which was completely overlooked so far. The corresponding values of ρ_c are large enough to initiate thermonuclear reactions, e.g. they are larger than ρ_c corresponding to M_{max} of $\alpha = 0$ case, whereas the respective core temperatures are expected to be similar. This explains the entire range of the observed over-luminous SNeIa mentioned above [1, 29].

Hence, based on a single underlying theory, i.e. a modified Einstein's gravity, varying the value of α , we obtain a range of sub- to super-Chandrasekhar limiting masses. From Fig. 3(b) it is clear that effectively α is a density dependent parameter, and, hence, brings in the chameleon-like effect (see, e.g., [30]) in the model. In a more rigorous model, the quantity equivalent to $|\alpha\rho|$ could be an invariant quantity (see [25] for details). Nevertheless, the main message here is that the modified gravity effect has a significant impact in white dwarfs.

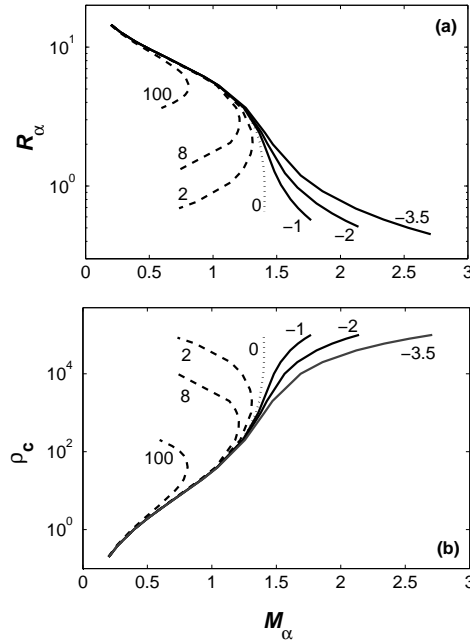


Figure 3: (a) Mass-radius relations. (b) Variation of ρ_c with M_α . The numbers adjacent to the various lines denote $\alpha/(10^{13} \text{ cm}^2)$. ρ_c , M_α and R_α are in units of 10^6 gm/cc , M_\odot and 1000 km respectively.

6 Summary and Conclusion

We have shown, by developing systematically the simplistic to rigorous models, that highly super-Chandrasekhar, as well as highly sub-Chandrasekhar, limiting mass white dwarfs are possible to exist. While the effects of magnetic field render only highly super-Chandrasekhar mass-limit(s), the effects of modified Einstein's gravity reveal sub- and super- both the limits and, hence, apparently unify two disjoint classes of SNeIa.

The new generic mass-limit of highly magnetized white dwarfs is in the range $2.6 - 3.4 M_\odot$, depending on the field profiles. Once the super-Chandrasekhar limiting mass is approached, the white dwarfs explode exhibiting over-luminous, peculiar SNeIa. Indeed observations suggest the exploding mass to be in the range $2.3 - 2.8 M_\odot$, which tallies with our theoretical calculation.

Now SNeIa are used as a standard candle in order to understand the size, and, hence expansion history of Universe, due the uniform mass of their progenitors. Now this 'uniform mass' will no longer remain uniform if the progenitor mass-limit is different — possible second standard candle — as we have established to be the case for certain magnetized white dwarfs. If the peculiar SNeIa are eventually observed

enormous in number, then it might necessarily need to sample the observed data from supernova explosions carefully which may affect the conclusion for expansion history of Universe.

Overall, our discovery raises two fundamental questions. Is the Chandrasekhar-limit unique? Is Einstein's gravity the ultimate theory for understanding astronomical phenomena? Both the answers appear to be no!

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